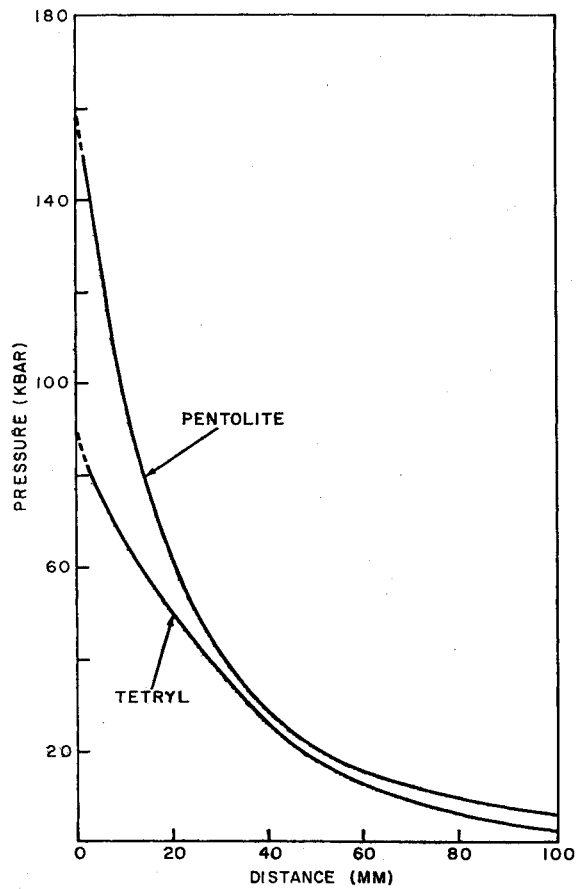

Fig. 2 Pressure vs gap

Table 3 Pentolite vs tetryl: shock sensitivity

Material	Donor	Gap, 50% point	Pressure, kbar	Mean, kbar
comp. B-3 (cast)	tetryl	209	16.4	
	pentolite	209	18.0	17.2
nitroguanidine	tetryl	46	63.0	
= 1.59 g/cm³	pentolite	53	83.2	73.1
nitroguanidine/wax	tetryl	16	78.8	
95/5	pentolite	25	119.7	99.3
= 1.55 g/cm³				

smaller gaps, agreement between the donors is not obtained because the calibration curves in this region are inaccurate, or because the pressure-time loading curves (not measured) affect the result, or because both of these factors are operative. As one approaches zero gap the pressure-time histories of the two donors should differ, and this factor probably has a major effect on inducing detonation of the acceptor. In other words, at the highest pressures, pressure amplitude alone does not define the shock sufficiently.

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Calculation of Damped Linear Systems by Holzer's Method

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HOLZER's tabular method for the calculation of torsional and other discrete linear systems is very well known. When treating the free vibrations by this method, i.e., when evaluating the natural frequencies, it generally is stated in the literature that the parameter of the Holzer table is the frequency ω.

The fact is, however, that in the case of free vibrations the parameter of the Holzer table is the eigenvalue Λ rather than the frequency ω.

To substantiate this statement, consider first the standard single-degree-of-freedom system that is damped both externally and internally. In this system, $(-s\dot{\varphi})$ = external damping torque (damping on the mass), and $(-u\dot{\varphi})$ = internal damping torque (damping parallel to the spring).

The differential equation is

$$-J\ddot{\varphi} - s\dot{\varphi} - u\dot{\varphi} - k\varphi = 0 \tag{1}$$

Substitution of the solution assumption $\varphi = \Phi e^{\Lambda t}$ in Eq. (1) gives the eigenvalue $\Lambda_{1,2}$ in the form

$$\Lambda_{1,2} = -[(s+u)/2J] \pm j\{\omega_v^2 - [(s+u)/2J]^2\}^{1/2} \qquad j = (-1)^{1/2} \tag{2}$$

hence in the form $\Lambda_{1,2} = -h \pm j\omega_D$. ($\omega_D$ = damped natural frequency; ω_v = undamped natural frequency.)

Generalizing this for the n mass system, one can say that, in the case of the solution assumption $\varphi = \Phi e^{\Lambda t}$, the eigenvalues of a linear system are of the form ${}_m\Lambda_{1,2} = -({}_mh) \pm j({}_m\omega_D)$. (Subscript m refers to the mth mode.)

Now, consider a multimass system damped both externally and internally. If the system is undamped ($s_i = u_i = 0$), the differential equation for the first mass reads

$$-J_1\ddot{\varphi}_1 - k_1(\varphi_1 - \varphi_2) = 0 \tag{3}$$

For the undamped system the "one-phase" solution assumption $\varphi_i = \Phi_i \sin\omega t$ is acceptable because there is no phase shift between the masses. Substituting it in Eq. (3), one has

$$\Phi_2 = \Phi_1 - (J_1\omega^2/k_1)\Phi_1 \tag{4}$$

which, as is well known, is the relation upon which the *ordinary* Holzer table for the undamped system is based.

If the multimass system considered is damped, then the differential equation for the first mass reads

$$-J_1\ddot{\varphi}_1 - s_1\dot{\varphi}_1 - u_1(\dot{\varphi}_1 - \dot{\varphi}_2) - k_1(\varphi_1 - \varphi_2) = 0 \tag{5}$$

Now a "two-phase" solution, hence either $\varphi_i = \Phi_i e^{\Lambda t}$ or $\varphi_i = e^{j\Lambda t}$, must be accepted because the damping produces a phase shift.

Upon substituting the solution assumption $\varphi_i = \Phi_i e^{\Lambda t}$ in Eq. (5), one obtains

$$\Phi_2 = \Phi_1 - [(-J_1\Lambda^2 - s_1\Lambda)/(k_1 + u_1\Lambda)]\Phi_1 \tag{6}$$

Comparing Eqs. (4) and (6), one sees that $J_1\omega^2$ and k_1 with the undamped system correspond to $-J_1\Lambda^2 - s_1\Lambda$ and $k_1 +$

Received by IAS August 6, 1962; revision received August 13, 1962.

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$u_1\Lambda$, respectively, with the damped system. Hence, the damped system can be solved by Holzer's Table 1, which will be called the *simple* (damped) Holzer table (because there is no imaginary number j in its heading) and for which the parameter of the table is necessarily the eigenvalue in the form $\Lambda_{1,2} = -h \pm j\omega_D$.

If the steps followed for the solution assumption $\varphi = e^{\Lambda t}$ are performed for the solution assumption $\varphi = e^{j\Lambda t}$, Holzer's Table 2 will be obtained ultimately. It will be called the *complex* (damped) Holzer table; its parameter is the eigenvalue in the form $\Lambda_{1,2} = jh \pm \omega_D$.† In the case of the undamped system, $s_i = u_i = 0$ in the tables and $h = 0$ in the eigenvalue. If in this case the eigenvalues $\Lambda = \pm j\omega_v$ and $\Lambda = \pm\omega_v$ are applied to Tables 1 and 2, respectively, the headings of both of these tables will be reduced to the heading of the *ordinary* Holzer table for undamped systems.

Table 1 Simple Holzer table for the calculation of damped linear systems; parameters: a) for free vibrations, $\Lambda = -h \pm j\omega_D$, and b) for forced vibrations, $(\Lambda) = j\Omega$

J_i	$-J_i\Lambda^2 - s_i\Lambda$	Φ_i	$(-J_i\Lambda^2 - s_i\Lambda)\Phi_i$	$\Sigma(-J_i\Lambda^2 - s_i\Lambda)\Phi_i$	$k_i + u_i\Lambda$	$\frac{\Sigma(-J_i\Lambda^2 - s_i\Lambda)\Phi_i}{(k_i + u_i\Lambda)}$
1	2	3	4 = 2 x 3	5	6	7 = 5/6

Thus, the statement about the eigenvalue as the parameter of the Holzer table in the case of free vibrations has been proved. It holds *generally*, i.e., for both damped and undamped systems (in the case of free vibrations). However, with the undamped systems both the eigenvalue and the tables themselves degenerate ($s_i = u_i = 0$ so that $h = 0$ and instead of ω_D there is ω_v). Consequently, in this special case of the general case the frequency may be (only practically!) considered as the parameter of the table. Theoretically, the parameter of the Holzer table in the case of free vibrations is only the eigenvalue irrespective of whether the system is damped or undamped.

Numerical Example

It is required that the eigenvalues of the first oscillatory mode ${}_1\Lambda_{1,2}$ be calculated for the torsional system of the following characteristics: $J_1 = 1, J_2 = 2, J_3 = 3$ (lb-in.-sec^2), $s_1 = 0.12, s_2 = 0.20, s_3 = 0.32$ (lb-in.-sec.), $k_1 = \frac{1}{3}, k_2 = \frac{1}{2}$ (lb-in./rad), $u_1 = 0, u_2 = 0.04$ (lb-in.-sec).

Table 2 Complex Holzer table for the calculation of damped linear systems; parameters: a) for free vibrations, $\Lambda = jh \pm \omega_D$, and b) for forced vibrations, $(\Lambda) = \Omega$

J_i	$J_i\Lambda^2 - j\,s_i\Lambda$	Φ_i	$(J_i\Lambda^2 - j\,s_i\Lambda)\Phi_i$	$\Sigma(J_i\Lambda^2 - j\,s_i\Lambda)\Phi_i$	$k_i + ju_i\Lambda$	$\frac{\Sigma(J_i\Lambda^2 - j\,s_i\Lambda)\Phi_i}{(k_i + j\,u_i\Lambda)}$
1	2	3	4 = 2 x 3	5	6	7 = 5/6

The eigenvalues of the system can be evaluated by the trial-and-error procedure by means of either Table 1 or Table 2 (assumption of h and ω_D, i.e., of the eigenvalue Λ in correct form for the given table, calculation of the table, and drawing of the remainder-torque curve until the remainder torque becomes zero).

The resulting simple Holzer table for the eigenvalue of the first oscillatory mode ${}_1\Lambda_{1,2} = -0.0626773 \pm j\ 0.4960997$ is shown here as Table 3. (Hence, the damped natural frequency of the first mode is ${}_1\omega_D = 0.4960997$ sec^{-1}; the undamped natural frequency of the first mode, calculated separately, is ${}_1\omega_v = 0.5$ sec^{-1}.) It should be noticed that Table 3 for eigenvalue is "closed" (remainder torque is zero) as it has to be.

† Practically, the numerical values in both simple and complex Holzer tables are the same and complex because the parameters are complex in both cases.

Table 3 Simple Holzer table calculated for ${}_1\Lambda_{1,2} = -0.0626773 \pm j\ 0.4960997$ (see numerical example)

J_i	$-J_i\Lambda^2 - s_i\Lambda$	Φ_i	$(-J_i\Lambda^2 - s_i\Lambda)\Phi_i$	$\Sigma(-J_i\Lambda^2 - s_i\Lambda)\Phi_i$	$k_i + u_i\Lambda$	$\frac{\Sigma(-J_i\Lambda^2 - s_i\Lambda)\Phi_i}{(k_i + u_i\Lambda)}$
1	0.2497078 + j 0.0026564	1.0	0.2497078 + j 0.0026564	0.2497078 + j 0.0026564	1/3	0.7491233 + j 0.0079693
2	0.4969084 + j 0.0251568	0.2508767 − j0.0079693	0.1248632 + j 0.0023513	0.3745710 + j 0.0050077	0.4974929 + j 0.0198440	0.7521221 − j 0.0199348
3	0.7466162 + j 0.0278133	−0.5012454 + j 0.0119655	−0.3745710 − j 0.0050077	0.0000000 + j 0.0000000		

In the case of forced vibrations of damped systems, the headings of both simple and complex Holzer tables remain as they are. In principle, the corresponding forms of the parameters hold in this case as well, except for $h = 0$. Thus, the simple Holzer table has to be calculated with the forcing frequency in the form $j\Omega$ and the complex one with that frequency in the form Ω, where Ω = forcing frequency.‡

The use of the damped-system Holzer tables for the calculation of both eigenvalues and eigenvectors of damped systems has been discussed fully in Refs. 1 and 2.

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‡ Hence, the incomplete view (that of the frequency as parameter of the Holzer table) is admissible in the case of forced vibrations calculated by the complex Holzer table. In all other cases (forced vibrations by the simple Holzer table, free vibrations by both tables), it induces the adoption of wrong procedures. On the other hand, only the eigenvalues of the simple Holzer table conform with the usual assumption that the "imaginary part of the eigenvalue is the natural frequency."

Systematic Matrix Calculation of Similarity Numbers

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BECAUSE the dimensions of any macroscopic variable u_j are of the form

$$\{u_j\} = D_1^{a_{1j}} D_2^{a_{2j}} D_3^{a_{3j}} D_4^{a_{4j}} \tag{1}$$

where D_1 is mass, D_2 length, D_3 time, D_4 temperature, and a_{ij} a positive or negative integer or fraction, the dimensions of a set of variables $u_1, \ldots u_n$ may be represented by a dimensional matrix $\|a_{ij}\|$, where i indicates the dimension or row number and j indicates the variable or column number. By definition, a similarity number is a nondimensional product of variables of the form

$$N = u_1^{e_1} u_2^{e_2} \ldots u_n^{e_n} \tag{2}$$

Received December 17, 1962.

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